

**Real Estate  
Accounting and  
Mathematics**

# Real Estate Accounting and Mathematics

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## Course Description

Real Estate Accounting and Mathematics is a refresher course covering the basic concepts and tools of accounting, finance, and math necessary for all real estate professionals. It is also applicable to accountants and as well as real estate investors. Topics covered are balance sheet, income statement, basic accounting concepts, real estate algebra, finance math, and more.

<b>Field of Study</b>	Specialized Knowledge - Real Estate
<b>Level of Knowledge</b>	Basic to Intermediate
<b>Prerequisite</b>	Basic Accounting
<b>Advanced Preparation</b>	None

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# Chapter 1:

## Real Estate Accounting

### Learning Objectives:

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After studying this chapter you will be able to:

1. Recognize basic financial statements and accounting procedures.
  2. Identify and apply different financial analysis ratios
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In general, the numbers we use in real estate finance represent dollars in the final result. To get there, we may use numbers as mathematical tools. We find it convenient, for example, to work with fractions and percentages. But before your anxiety index gets out of control, be assured that we will not get too involved in this chapter in fancy computations. Rather, we'll concentrate on very straightforward calculations used in everyday real estate practice.

In real estate finance, we work with numbers in financial statements. The processes and procedures used to record measures of dollars and cents, according to categories, are called *bookkeeping*. The orderly arrangement, display, and analysis of these numbers are termed *accounting*. Probably the most important information used in the world of finance is that presented in financial statements. We'll examine the basic principles of accounting and look at some elementary analysis of these essential documents.

The first section will be devoted to the accounting material and the second section to mathematics of finance.

# Basic Accounting

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The basic function of accounting or bookkeeping is to maintain a record of financial transactions. From these records, an accountant can prepare a statement that shows the aggregate result of these transactions for a period of time. From such a statement we can see whether we are ahead or behind, i.e., whether we have made a profit or suffered a loss. We call it a *profit and loss* or *income statement*. This statement displays the totals of income and expenses by categories of transactions.

But while we may want to know the results of financial transactions occurring during a specific time period, we also need to know our overall position at any instant in time. For that we have a *balance sheet* or a *statement of financial position*. Taken together, an income statement and a balance sheet will constitute a set of financial statements. These statements are prepared from the records of money transactions.

The basic record in a set of books is called the *ledger* account. Physically, it is generally a piece of paper divided down the middle. The expression T-account suggests how it looks. Separate accounts are established for each of the categories of income, expenses, assets, and liabilities. An important part of bookkeeping and accounting education involves learning the rules for classifying and setting up the basic accounts. A *chart of accounts* is a listing of all account titles used within an accounting system. Business transactions affecting these accounts are initially recorded by journal entries and then posted to the individual accounts maintained in the ledger.

By custom, the numbers for things we own, called *assets*, are recorded on the left side. The numbers for what we owe, called *liabilities*, are recorded on the right. An entry on the left is called a debit and on the right, a credit. If we looked at the ledger account for cash in the bank, we would see the record of deposits on the left side and withdrawals on the right. The balance, being the difference between the two, assuming we were not overdrawn at the bank, would appear on the left. The money we have in the bank is one of our assets.

The expression *double-entry system* is used to indicate that for each transaction, a record has been made in at least two places. This is not a duplication or carbon copy. Rather, it is done to provide more adequate information. For example, assume that we are setting up the books for a real estate office. We'll need a ledger account for cash in the bank, one for commission income, one for advertising expenses, etc. In the income and expense ledger accounts, entries for funds received will be made on the right and for money paid out, will be on the left. These must balance with the entries made in the account for the bank. When we receive a commission check from the title company and deposit it in our bank account, we would enter the amount as a debit in the bank account and as a credit in the commissions account. The sum of debits and credits for each transaction must be equal. Here we've made a double entry in the records. Later we can copy the balances from each ledger account and prepare a financial statement. There will also be more steps to take, called journal entries, to make adjustments according to accounting practice.

The process of keeping the most simple set of records, containing only information concerning actual cash coming in and going out, is known as *cash basis accounting*. When allowance is made in the records for noncash items, such as depreciation, and for apportioning income or expenses over time, we designate the method as *accrual basis accounting*. Most individuals, to the extent they have records at all, operate on a cash basis. Most businesses use the accrual method. We will examine both types.

Obviously, accounting records are maintained and statements are prepared for many uses in no manner associated with real estate finance. It is well to realize that accounting rules and procedures are equally useful whether the statements are to be used in analyzing, investment in real estate, obtaining a mortgage, or showing the financial results of a business to stockholders or the IRS. It is the universal acceptance for accounting methods and statements prepared according to them that makes it important for everyone to have some understanding of accounting. What we describe here therefore will be applicable to any financial statement regardless of the use to which it may be put.

There is not a lot of mystery involved in how financial records should be kept. For individuals, about all that is needed is a bank checkbook that has been correctly entered for each deposit and withdrawal, along with invoices and receipts. For a business, the same fundamental activity goes on but in a much greater volume. Significant education, however, is required to learn how to record certain transactions and, above all, to prepare financial statements. In the business world, in order to be certain that a financial statement has been prepared according to established accounting principles, lenders and others may insist that all statements to be considered in conjunction with credit extension be certified. This is where the CPA (certified public accountant) comes in.

When a CPA has prepared a financial statement, a certification will appear on it to the effect that it conforms to generally accepted accounting principles. Anyone expecting to rely on the financial data presented will look carefully to see whether there are any exceptions stated in the CPA's certification. As a general rule, when a request is made for a certified statement, only one prepared by a CPA without qualification in the certification will be acceptable.

None of the foregoing means that statements prepared by non-CPAs are unsatisfactory. Probably the majority of financial statements prepared in the country, as a whole, are not certified. Many small businesses and almost all individuals do not choose to use the professional services of a CPA. (This is not to say they may not need professional help, however.) The most important consideration is: Does a set of financial statements accurately portray the financial condition of the entity? The CPA's certification attests to his professional skills, and people can rely on them. A noncertified statement, however, can be just as accurate.



# Financial Statements for Individuals

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We discussed the typical financial information required by a lender from an applicant for a home mortgage. We'll now take another look at what is involved.

For a family unit, an operating statement would consist of a listing of income together with the amounts and nature of monthly or periodic expenditures. To get the flavor of how accounting procedures would be applied to an individual, let's consider a profit and loss or income statement prepared in a formal manner.

## Income Statement

The basic function of this statement is to show the totals of all income and outgo for a period of time according to customary categories. By examining it, a person can see whether there is a profit or a loss for that particular time. The timing of the receipt of income or the payment of expenses is very important, especially if the cash basis is used.

**Some observations:** It is unlikely that there is any family, anywhere, which has prepared an income statement in this form, even a CPA's family. But we're using it to gain an understanding of the concept of the income statement in an accounting sense and how it is used in business. Note that the gross income is shown as revenue and the taxes and other items withheld appear as deductions. Further, the income tax refund on the prior year's tax return shows as revenue, simply because it was cash received during the current reporting period.

Jack K. and Jane Q. Smith			
INCOME STATEMENT			
for the year ended December 31,20A			
Revenue:			
John's earnings	\$18,000		
Mary's earnings	12,000		
Interest income	150		
Income tax refund	500	\$30,650	
Revenue Deductions:			
State income tax withheld	400		
State disability ins. taxes	115		
Health insurance premium	700		
Life insurance	300		
Car insurance	400		

Homeowner's insurance	200	
Mortgage payments	3,600	
Property taxes	950	
Household expenses	8,500	
Clothing	1,100	
Entertainment	600	
Automobile expense	1,200	
Car payments	880	
Utilities	750	
Vacation	1,000	
Maintenance and repairs	1,300	
Gifts and church contribution	450	
Savings account deposits	1,200	
Miscellaneous expenses	500	28,945
Net Revenue		\$1,705

Because this reflects cash basis accounting, there is no provision for either additional tax liability or a refund applicable to the current year. Nor is there any provision for debts incurred before the end of this year and payable next year. The Smith's might have made several purchases in December and, in fact, owes a large sum to be paid in January. Cash basis accounting does not provide for showing such a situation in financial statements.

Of course, a CPA would probably object to preparing an income statement on a cash basis. Instead, he would want to include an allowance for income earned but not yet paid. When those items are included, along with a few other adjustments, the net revenue would then fairly reflect the financial progress (or regression) for the period. For the individual, these items may not matter much, but for business they do.

Finally, what about the apparent profit of \$1,705? If our record keeping has been complete and accurate, this indicates that we have \$1,705 more cash at the end of the year than we had at the beginning. And that's all it means. It certainly should not be interpreted as a profit or surplus unless our outstanding debts and the value of our assets are exactly the same at the end as they were at the beginning of the accounting period. And that introduces us to the need for the other financial statement: the balance sheet.

## The Balance Sheet

The primary function of this financial statement is to show the values of assets and liabilities as of a single point in time. The difference between these two amounts is the entity's net worth. Let's look at how a statement would show the financial condition of the Smith family.

Jack K. and Jane Q. Smith

### BALANCE SHEET

as of December 31, 20A

ASSETS			
Current Assets			
Cash on hand and in bank	\$ 750		
Savings accounts	3,500		
Cash value of life insurance	1,200	\$ 5,450	
Other Assets			
Automobile	\$ 6,000		
Personal property	20,000		
Real estate (market value)	110,000	\$136,000	
TOTAL ASSETS			141,450
LIABILITIES AND NET WORTH			
Current Liabilities			
Accounts payable	\$ 850		
Car payments	110	\$ 960	
Other Liabilities			
Mortgage on home—balance	23,500		
Car loan—balance	2,000	25,500	
TOTAL LIABILITIES			26,460
Net Worth			114,990
TOTAL LIABILITIES AND NET WORTH			141,450

**Some observations:** What do we really mean by net worth? If our numbers are correct representations, we might say that this family entity could dispose of its assets, pay its debts, and have \$114,990 in cash left over. This assumes, however, that all the assets could, in fact, be turned into cash in the amounts used in the statement. This is a key item in any balance sheet. How realistic are the values placed on the assets?

While the basic function of the balance sheet is to show the net worth, that number is only as good or valid as the numbers used for the components. If assets are overvalued and liabilities are understated, the net worth figure is meaningless. To overcome this and other potential difficulties, accountants are expected to follow specified uniform procedures in arriving at the numbers to be shown in financial statements. It's not the figures that lie, it's the liars who figure.

**Summary.** In these highly oversimplified examples, we've seen the essential structure of income statements and balance sheets. It is emphasized that formal accounting statements are prepared as a rule only for businesses and for individuals with diverse and extensive financial interests, not for the typical homebuyer. In the next section we'll study the statements drawn from records maintained on an accrual basis as is customary in business.

## Chapter 1 Review Questions – Section 1

1. The basic financial statements include the balance sheet, income statement, statement of cash flows, statement of retained earnings, and statement of changes in retained earnings. True or False?

2. The primary purpose of the balance sheet is to reflect

- A. The fair value of your assets and liabilities at some moment in time.
- B. The status of your assets in case of forced liquidation.
- C. Assets, liabilities, and equity.
- D. Your assets' potential for growth.

3. Bookkeeping is:

- A. The processes and procedures used to record measures of dollars and cents, according to categories.
- B. The orderly arrangement, display, and analysis of financial transactions.
- C. Analysis of financial statements.
- D. Compliance with financial accounting standards.

4. Balance sheet is also called:

- A. An income statement.
- B. A statement of financial position.
- C. A financial analysis statement.
- D. A profit or loss statement.

5. Business managers want to know the operating results of financial transactions occurring during a specific time period as well as business' overall position at any instant in time. True or False?

6. The term "double-entry system" refers to

- A. The use of real and nominal accounts.
- B. The recording of each transaction in two parts.
- C. The use of two journals.
- D. The use of a journal and a ledger.

7. A chart of accounts is

- A. A flowchart of all transactions.
- B. An accounting procedures manual.
- C. A journal.
- D. A list of names of all account titles.

# Financial Statements for Business

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The real estate practitioner is authorized to negotiate the sale of a going business, so she should know something about how to evaluate one. The technical details of the process of evaluation are beyond the scope of this text. Practices vary from one kind of business to another, but in every case, the starting point is the accounting records. We will examine how financial statements may be used in determining the value of a business.

Because financing often is required in the sale and purchase of a business, we must also consider how income statements and balance sheets for going businesses are used by lenders. Our treatment of these important matters will be brief and general. There is a great deal more to know than can be presented here.

Assume that we're looking at a neighborhood retail hardware store. The owner of the business is interested in selling both the business and the real estate, and we hope to find a buyer and complete the transaction. We'll need financial statements for starters.

## Accounting Rules.

With respect to these two financial statements, we'll consider some of the rules followed by accountants. Knowledge of these rules is essential for an understanding of the meaning of the numbers that appear. Let us look at the balance sheet, or statement of financial position.

A current asset is one that is in the form of cash or can be expected to be turned into cash during the 12 months. Equipment, buildings, and land are long-term or fixed assets even though the owner might plan to liquidate them in the near future. A current liability is an obligation payable within the next 12 months. An important measure of the solvency of a business is the difference between the current assets and the current liabilities. For most businesses, this relationship, called the current ratio, should be not less than 2:1. That is, the current assets should be at least twice the current liabilities. In the event a financial crisis develops, a business should be able to pay its current obligations out of its current assets and have some to spare. If the current liabilities exceed the current assets, the firm is in financial difficulty.

The inventory is to be valued at the lower of cost or market value. As a rule, the retail value of the inventory will exceed its cost--that's the whole idea of retailing. Buy low and sell high. But if the stock becomes obsolete and worth very little, that condition should be reflected in the financial statement. The buyer of a business who is taking over the stock of merchandise will need to check carefully to see how saleable the stock actually is and how its value relates to that shown on a financial statement.

Here we see the impact of the accrual accounting method. Note that there is an accounts receivable. The merchandise is gone, and the amount of the sale is included in the total of net sales. In terms of the business activity, the value of the credit sale is to be included and profit is taken. If, of course, a receivable becomes a bad debt, a charge against profit will be made later. The prepaid expense item is shown to reflect that some foods or services were purchased but not fully used during the accounting period. For example, insurance is paid for in advance. If an accurate computation of business profits is to be made, only the proportionate cost of the insurance (or supplies, etc.) actually applicable to the time period should be charged. The balance of the expenditure is a prepaid expense and is counted as an asset.

The Real Value Hardware Store, Inc.

BALANCE SHEET

DECEMBER 31, 20A

ASSETS

Current Assets

Cash on hand	\$ 900	
Cash in bank	8,800	
Accounts receivable	900	
Merchandise inventory	200,000	
Prepaid expenses	1,400	
Total current assets		\$212,000

Property, Plant, and Equipment

Land	\$ 80,000	
Building	200,000	
Equipment	8,000	
Automobile and trucks	6,000	
Furniture and fixtures	5,000	
	\$299,000	
Less accumulated depreciation	65,000	
Net property		234,000

TOTAL ASSETS \$446,000

LIABILITIES AND STOCKHOLDER'S

EQUITY

Current Liabilities

Trade accounts payable	\$ 33,000	
Salaries, payroll taxes	8,500	
Other taxes	5,000	



Current portion long-term debt	12,000	
Total current liabilities		58,500
Long-Term Debt		
Notes payable to bank	\$100,000	
Less current portion	12,000	
Total long-term debt		88,000
TOTAL LIABILITIES		146,500
Shareholders' Equity		
Common stock	\$250,000	
Retained earnings	49,500	
Total shareholders' equity		299,500
TOTAL LIABILITIES AND STOCKHOLDER'S EQUITY		\$446,000

**INCOME STATEMENT**  
for the Year Ended December 31, 20A

Revenue			
Gross sales		\$750,000	
Cost of goods sold		450,000	
Gross profit on sales		\$300,000	
General, selling, and administrative expense		240,000	
Net operating income		\$ 60,000	
Other income			
Discounts earned		1,500	
Gain on sale of equipment		3,000	\$64,500
Revenue Deductions			
Interest expense		\$ 8,000	
Bad debts		500	8,500
Income Before Income Taxes		\$56,000	
Provision for federal and state taxes		32,000	
Net Income After Tax		\$24,000	

The values shown for assets such as real estate and equipment will be recorded at cost. It is customary to show first the original book value (cost) and then deduct the total depreciation charged to the date of the statement. The alternative is to show only the current book value, but it is considered more informative to show both original cost and accumulated depreciation as is done in our example.

Under current liabilities, all obligations to be met within the next 12 months are to be shown. In the case of an installment debt or mortgage, the current portion would be simply 12 times the monthly payment, for example. Note that the payment for both principal and interest is considered even though, as we'll see later, only the interest expense is considered when computing taxable income. It is customary to show the total amount of the debt outstanding as of the date of the statement less the portion treated as current.

You will have noted that we are dealing with a corporation. The ownership of the company is represented by shares of stock. The book value of the shares is determined by the total of the

shareholders' equity. This consists of the original amount paid in to the corporation by the purchasers of the stock plus the portion of the firm's earnings in the past, which were not distributed as dividends and were therefore retained in the business. If we assume that there were 1,000 shares of stock issued, then each share would have a book value of  $299,500/1,000 = \$299.50$ . Is this the real value of these shares?

Notice that the amount of the shareholders' equity is arrived at by subtracting the sum of the liabilities from the total assets. In theory, all the assets could be turned into cash for the total shown, and if the debts were completely liquidated, the amount left over would be equal to the shareholders' equity. This is the net worth of the firm. It should be clear that if the assets have been incorrectly valued or some liabilities omitted, this net worth would not be realistic. The net worth, expressed in terms of book value of shares outstanding, is a useful measure but is by no means a value a buyer would pay without considering the matter much further.

The net worth figure is useful when comparisons are made from one accounting period to another. In general, if the net worth is increasing, it is considered that the firm is prospering. But there are other things to consider. What about the distribution of earnings to the owners? Where do they show?

The change in the retained earnings item from one year to the next is determined by the amount of net income generated and whether a dividend was paid to the stockholders. In our example, the \$49,500 represents the accumulation from the beginning of the business. We're not told here about the dividends. Looking ahead for the moment, note that the income statement shows net income after taxes of \$24,000. If we assume that the board of directors of the company decided to declare a dividend of \$14 per share, the total dividend payout would be \$14,000. From that information, we could conclude that the total shareholders' equity (net worth) at the end of the previous year was \$10,000 smaller. To explain, consider the following customary reconciliation included in statements of financial condition.

#### Reconciliation of Retained Earnings

Balance at end of previous period	\$39,500
Add net income	24,000
	\$63,500
Less dividends paid	<u>14,000</u>
Balance at end of current year	<u>\$49,500</u>

## Financial Analysis

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So we come back to the basic questions: what is this business worth? What should the selling price be? Is its financial condition healthy? Should it be granted a loan?

The purchase of a going business, and particularly one in which the buyer is to be active, requires a careful evaluation of both the value of tangible assets and the earning ability of the business. This latter is often referred to as goodwill. Obviously, it may be presumed that at least some of the customers will continue to come to the store so some sales will be made right from the day of takeover by a new owner. In contrast, it may take quite a while before a brand new business attracts enough customers to generate any significant sales volume. The difference between the market value of the tangible assets (such as the real estate, inventory, and equipment) and the purchase price of a business represents the value placed on the profits to be generated from this continuing business - the goodwill.

The buyer is making an investment and expects a return. Here he could look solely at the net worth and offer to pay \$299,500 to step into the shoes of the present shareholders. But notice if the real estate, for example, had a market value of say \$500,000, the buyer on such a basis would probably be getting a real bargain. Net worth on a balance sheet is not ordinarily a satisfactory measure of value for the purpose of deciding on a selling price.

But what about the earnings and dividend? Can't they be used to establish a value? If you pay \$100 for a share of stock and the dividend is \$10 a year, you can say you are earning a 10 percent return. Once you decide on the rate of return you want, you quickly can calculate the price you would pay for a share of stock, given the amount of dividend. For example:

Dividends are \$14 per share per year

Desired yield is 10 percent

Price per share is  $\$14 \div .10 = \$140$

Proof: If you invest \$140 at 10 percent, the return would be \$14

In our example, if the proposed purchaser of this business wanted a 10 percent return, he would offer \$140 for each share of stock. If you were the seller, would you accept the offer?

You would receive  $\$140 \times 1,000$  shares for a total of \$140,000. According to the balance sheet, those shares are worth \$295,000. Would this be a good deal for the seller?

You might conclude (and you should) that setting the value on a going business is a complex matter.

What value can be placed in this case on the fact that the business is established and that there is reasonable expectation that it will continue to earn a substantial profit at an increasing rate? Here it could be argued that the cash dividend of \$14 per share is not the proper measure, but instead the \$24 per share of earnings should be used. For a 10 percent return, a purchaser may then be willing to pay \$240,000. (This is calculated as before.) But does this really adequately allow for the goodwill?

Before reaching any more conclusions about earnings and the selling price of a going business, let's take a closer look at the earnings or income statement. Refer again to the illustration.

The accounting rules applicable to the preparation of the income statement require that all the data for the accounting period indicated be used. Whereas a balance sheet utilizes values applicable at a single point in time, the income statement involves the aggregate of sales income and expenses occurring during a period of time. This may be a month; more commonly income statements cover a 12-month period. The balance sheet must show the figures as of the end of the income statement period to be of the most use. Here we have a balance sheet as of the end of July. The income statement is for the 12 months ending on the same date.

The nature of the individual items in an income statement will vary according to the kind of business. Here we find that expenses of all types are grouped. The wholesale cost of the merchandise is included in the cost of goods sold item. Payroll and other operating expenses are included in the general, selling, and administrative expense category. Within the firm's accounting records there will be separate ledger accounts for each individual type of expenditure. Totals are drawn together when preparing the financial statements.

One of the many important uses of financial statements is in the comparison of one period against another. Also, one firm can be compared against another or the industry. It is customary to convert income statement data into ratios or percentages to make this comparison easier. For example, could you express the gross profit as a percentage of sales? (You should get 40 percent here.) In turn, how about eight percent as the net operating income rate of return on sales? If a prospective buyer knew that retail hardware stores generally produced a net return on sales of, say, 15 percent, what conclusion might he draw about this business?

There is much more to ratio analysis than is shown here. But it should be clear that financial statements can (and must) be used to get a handle on the financial condition of a business. This is certainly true for lenders.

The prospective buyer may not have enough cash to satisfy the seller's down payment requirements. Can he borrow the deficiency? Where would he apply? Banks, rather than savings and loan associations, make loans to businesses. As a general rule, the bank prefers borrowers who are already in business and have demonstrated an ability to run a profitable operation. Loans to increase inventory are often readily available. The borrower is expected to repay the loan out of sales income, so it is a short-term proposition. Such working capital loans often are made for three or six months at a time, and the promissory notes may be renewed if all is going well. Loans to purchase equipment are also available (they may be repaid in periodic installments), and then there are real estate loans to buy business property.

In considering a loan to a going business, the lending officer will be interested in a third financial statement customarily prepared for business which shows the flow of cash. This statement, *cash flow statement*, provides a way to see where money came from and the uses to which it was applied during the accounting period.

DELTA COMPANY  
Cash Flow Statement  
(In Thousands of Dollars)  
For the Year Ended December 31, 20x7

**20X7**

<b>Cash flows from operating activities:</b>		
Net income	\$9.6	
Add (deduct) to reconcile net income to net cash flow		
Depreciation	3.0	
Increase in current liabilities	5.4	
Increase in accounts receivable	(5.0)	
Increase in inventory	(5.0)	(1.6)
Net cash flow from operating activities		8.0
<b>Cash flows from investing activities:</b>		
Cash paid to purchase marketable securities	(13.0)	
Cash paid to purchase fixed assets	(5.0)	
Net cash flow used for investing activities		(18.0)
<b>Cash flows from financing activities:</b>		
Issuance of long-term debt	5.0	
Net cash flow used in financing activities		5.0
Net decrease in cash and cash equivalents		(5.0)
Cash and cash equivalents at the beginning of the year		35.0
Cash and cash equivalents at the end of the year		\$30.0

Unless the prospective buyer of our hardware store has been in business and has established himself with a lender, it is unlikely that he would borrow funds on an unsecured basis to buy the business. It is possible that the real estate could be used as security, of course. In any event, a lending officer would require financial statements for the business to get an idea of its financial condition. The borrower would also be asked to provide complete financial data on himself. No financial information - no loan!

A final word on the valuation of a going business. Here we have been looking at a typical small business. Whether it is incorporated makes no basic difference. The purchaser must evaluate the tangible assets, and he must estimate future earnings based on the past. Much of the information required for these judgments comes from financial statements. In the final analysis, however, there is considerable guesswork involved in determining what price should be paid, and there is no substitute for experience.

If we're dealing with a large-scale operation, similar analytical steps are required but more use will be made of the price per share of stock, especially if it is listed on a major stock exchange. In effect, the marketplace has put a value on the business. Shrewd investors, however, may find cases where undervalued assets have been overlooked, and they can move to pick up a bargain. It is a fascinating business!

## Conclusion

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The accuracy of financial data is all-important because judgments made on the basis of false information may lead to disaster. The role of the real estate practitioner in the buying and selling of businesses is primarily to match buyers and sellers. He will advise the parties to obtain professional advice from accountants and attorneys for assistance in confirming the accuracy of information and guidance in entering into agreements.

## Chapter 1 Review Questions – Section 2

8. Accrual basis accounting only involves information about actual cash coming in and going out. True or False?

9. The best indication of an enterprise's present and continuing ability to generate favorable cash flows is information about enterprise earnings based on accrual basis of accounting. True or False?

10. What function do general ledgers serve in the accounting process?

- A. Reporting.
- B. Summarizing.
- C. Classifying.
- D. Recording.

11. Financial statements prepared by non-CPAs are:

- A. Unsatisfactory.
- B. Not certified.
- C. Certified.
- D. Not accurate.

12. Revenue deductions do NOT include:

- A. State income tax withheld.
- B. Income tax refund.
- C. Health insurance premium.
- D. State disability insurance taxes.



13. The balance sheet assets do NOT include:

- A. Cash on hand.
- B. Automobile.
- C. Real estate.
- D. Accounts payable.

14. Which of the following statement is incorrect?

- A. A current asset is one that is in the form of cash or can be expected to be turned into cash during the next year.
- B. A current liability is an obligation payable within the next 12 months.
- C. The solvency of a business is the difference between the current assets and the current liabilities.
- D. If the current assets exceed the current liabilities, the firm is in financial difficulty.

# Chapter 2:

## Mathematics of Finance

### Learning Objectives:

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After studying this chapter you will be able to:

1. Recognize and apply key financial ratios useful for real estate.
  2. Calculate interest payments, insurance premiums, loan values, and income ratios.
  3. Recognize and calculate various area measurements useful for real estate valuation.
- 

Even if mathematics is not your favorite, many revel in working with numbers. Society couldn't function without mathematics; computational tools are absolutely essential. Certainly, in real estate, we need ways to calculate interest, mortgage payments, and even prorations. Those who enjoy mathematics have seen how useful it is to be able to do this work, and they get a "bang" out of their achievement. It is quite all right, even respectable, to enjoy working with fractions, percentages, and prorating a tax bill.

In this section we will first review some very basic computational steps essential to all the mathematics we need in real estate finance. Then we'll examine how each of the usual calculations is made. Finally, you'll have an opportunity to test your understanding and get some practice.

### The Basic Use of Numbers

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There are four basic arithmetic steps: adding, subtracting, multiplying, and dividing. As a general rule, most arithmetic we encounter in the everyday business of real estate involves whole numbers. All it takes is practice (or a hand calculator) to become proficient. By whole numbers we mean, of course, such values as 5,315, 4,756, etc. When we need to make calculations involving fractions and

percentages, there are a few additional rules to follow. Assuming that no review of working with whole numbers is required, we'll move to fractions and percentages where some find a little mystery. We'll get rid of the mystery in a hurry.

### **Fractions**

By definition, a fraction is simply a number, representing a quantity that is smaller than one or a whole, and has two parts: a numerator and a denominator. For example:

$$1/2 \text{ and } 1/4$$

are fractions. The 1 is the numerator and the 2 and 4 are denominators. A denominator shows the number of equal parts into which a whole number has been divided. The numerator designates how many of those parts have been measured. The numerator is always on top and denominator on the bottom. (N over D.)

If the denominator is larger than the numerator, the value is in the form of a proper fraction. A fraction in which the numerator is larger than the denominator is called improper. One and a half months as a unit of time might be written as  $3/2$ ; this is an improper fraction. Of course, we could also designate the time period as  $1 \frac{1}{2}$  months. We refer to the  $1 \frac{1}{2}$  as a mixed number. In making calculations it sometimes simplifies matters if a fraction is changed to one of its other forms. It is for that reason we are concerned at all with these alternatives.

To calculate the total dollars of rent payable for a seven-week period, when the monthly rate is \$400, there are several choices:

$$400 \div 4 = \$100 \text{ per week}$$

and

$$7 * \$100 = \$700$$

Also,  $1/4$  of \$400 = \$100 per week, etc. Seven weeks may also be represented by  $(1/3)/4$  months. That form is awkward to work with, so we change from a mixed number to an improper fraction:  $7/4$ . Then  $7/4 \times \$400 = \$700$ .

Because there are twelve months in a year, a charge, such as for property taxes or insurance applied uniformly throughout the year, for one month would be calculated by first determining the fraction of the whole. The numerator would be 1 and the denominator would be 12. Hence we have  $1/12$ . This may also be written  $1/12$ .

If, instead, we were concerned with charges for three months, the fraction needed would be  $3/12$ .

Because we will be using the fraction in a multiplying relationship with the dollars of expense, to make the arithmetic easier, it is customary to simplify the fraction. Here the process is called reducing. Thus:

$$3/12 = 1/4$$

When we divide 3 into both numerator and denominator, we end up with 1/4.

Assuming that the taxes for the full year amount to \$960, the portion for a three-month period would be:

$$1/4 * \$960 = \$240$$

It is always good policy to check the result of a calculation. Here we can say that if one quarter equals \$240, then the whole would be 4 times that amount.

Thus  $4 \times \$240 = \$960$ . (Proved correct.)

Note that we assumed we were dealing with a full year's taxes. Because tax bills typically show two equal installment amounts we may work with the cost for six months. We have a choice when computing a proration. (Recall that prorating means simply to spread out equally.) We can find the charge for time periods related to half a year, or double the installment, and work with fractions of a whole year. Here it might be easy to treat the installment as an annual amount by mistake.

For an installment covering six months, a fraction can be expressed in sixths. Hence for two months the tax charge would be:

$$2/6 = 1/3 * \text{installment}$$

When the time period is in units of days, the fraction can be devised as follows. In most cases, it is assumed that each month has 30 days and therefore the year has 360. For a time period such as five months and eight days, the fraction would be:

$$5 * 30 = 150 + 8 = 158$$

Hence 158/360. The fraction would then be applied against the appropriate dollar amount for a whole year. Note that it is the denominator of the fraction that must correspond to the total time period for which the dollar amount applies.

So far we've considered only multiplying by fraction. In a later section dealing with finding unknowns, we'll see how fractions are used when dividing. Fractions, just as whole numbers, can be added and subtracted. That is not encountered very often in real estate finance, so we'll skip it here.

But what about decimals? Our money, for example, is measured according to the decimal system — units of 100. We can express any fraction in decimal form too; this is done largely for convenience in computing. Frequently it is preferred if the fraction is changed into a decimal equivalent before making

the calculation. There are some problems, however, because not every fraction can be expressed completely as a decimal.

For example, the fraction  $1/2 = .50$ . Half a dollar is clearly 50 cents. A dollar and a half is written \$1.50. Note therefore that the numbers to the right of the decimal stand for the fractional portion of the whole; the whole is 1. If taxes are to be charged for three months, the proportion would be calculated by using either  $3/12 = 1/4$  or  $.25$ . If the annual charge is \$1,200, the proration would amount to  $.25 \times \$1,200 = \$300$ .

The decimal of  $.50$  is calculated from  $1/2$ . By dividing the denominator into the numerator (2 into 1), the decimal equivalent is obtained. We know the result is less than one because the fraction was smaller than one. Therefore the decimal point must be placed to the left of the number obtained by dividing. Let's look then at the decimal equivalent of  $1/6$  which is needed when apportioning a charge over a six-month time period, for example.

The fraction of  $1/6$  calls for, in writing the decimal equivalent, the six to be divided into 1. (Try it— you may not like it) Note that there is always something left over. The result will be  $.16666\dots$ . The same condition will hold when dividing by 3 or any multiple. How about dividing by 7?

It turns out that sometimes it is better to work with the fraction rather than its decimal equivalent. Assume that the annual tax bill is payable in two installments of \$660 each. The proration for two months would be:

1.  $2/6 = 1/3; 1/3 * 660 = \$220$
2.  $2/6 = 1/3 = .3333$   
 $.3333 * \$660 = \$219.9999$

It is obvious from the problem that the correct answer is \$220. Of course, it can be argued that when the  $\$219.999\dots$  is rounded, the same result is reached. It doesn't always work out as easily.

If we borrow \$3,000 at 12 percent per annum for two and a half years, how much interest will we have to pay? Our concern here is with the two and a half years; we'll review interest computations shortly. It's probably self-evident that the time period could be written simply as  $2 \frac{1}{2}$ . If we had to multiply (or divide) by the  $2 \frac{1}{2}$  we could do so but probably not with ease. Here's another case where conversion to decimal form makes the job easier. Because the whole is one in the decimal system, then:

$$2 \frac{1}{2} = 2.5$$

As shown later, interest =  $\$3,000 * .12 * 2.5 = \$900$

The following generalization may apply: When working with fractional amounts and doing the computation by hand, regular fractions will probably be easier. If a calculator is used, the fraction will have to be converted to its decimal equivalent. Always prove the answer.

Decimal expressions bring to mind percentages.

## ***Percentages***

Once again we deal with parts of a whole. We may say that expenses amount to 40 percent of rental income; sales commission will be 7 percent of the selling price; or the fire insurance policy has an unearned premium amounting to 55 percent. We also can say that the income is 115 percent over last year.

The whole of anything is 100 percent. If you have less than a whole, the measure will be shown as a number less than 100. While speaking in percentage terms is convenient and conveys our meaning adequately, we must change any percentage expression to either decimal or fractional form if we are going to perform a computation.

In some cases, we know what the percentage is. We decide that we wish to earn 7 percent commission or the rate of interest is given at 5.25 percent. In others, we want to express a relationship in percentage terms. Percentage expressions are particularly useful in making comparisons. An investment in property A will yield a return of 9 percent on invested funds, whereas property B is expected to produce 11 percent. If we are concerned primarily with rates of return rather than dollars of return, we will compare yields in percentage form.

Assume that an investment of \$10,000 cash yields income after taxes of \$750 or that we could invest \$50,000 to generate a return of \$8,000. The yields would be computed as follows:

$$\$750/\$10,000 = .075$$

$$\$8,000/\$50,000 = .16$$

That's the first step - obtain the decimal form for the fraction formed by income over investment. To obtain the percentage form, simply multiply by 100 percent.

$$.075 * 100\% = 7.5\%$$

$$\text{and } .16 * 100\% = 16\%$$

*Note the following rule: when multiplying by 100 percent, move the decimal point two places to the right and add the percent sign (%).*

Here we have created the percentage form in order to communicate a measure useful in comparisons. Perhaps even more often we must use a percentage expression in making a computation.

The real estate commission is to be 7 percent on the sales price. What is the commission in dollars? Consider a sales price of \$112,500.

$$7\%/100\% = .07$$

$$.07 * \$112,500 = \$7875$$

*Note the two-step sequence. First, divide the percentage by 100 percent. Earlier we multiplied a fraction by 100 percent to obtain the percentage. To reverse the process, we divide. If you think about it, dividing is just the reverse of multiplying, and vice versa.*

Next the decimal is applied against the whole amount. What we are really looking for is 7/100ths of the sales price.  $7/100 = .07$ . In multiplying, the x is a symbol for "times". It may also be read as "of." Try it.

Now let's assume that we forecast that rental income will be 112 percent of last year's amounts. We look at our records to see that the income was \$15,000. How much will this be increased? We see that the increase is simply 12 percent, so:

$$12\% = .12$$

$$.12 * \$15,000 = \$1,800$$

$$\text{The new value would be } \$15,000 + \$1,800 = \$16,800$$

In working with decimals, it obviously is important to get the decimal point in the right place. After every conversion, you should check back to see if your result is reasonable. An easy way to do this is to think in terms of money. For example, three-fourths in decimal form is .75. Is that the way you would write seventy-five cents? It had better be. If you increase your funds on hand by one half, can you say the new amount is 150 percent of the former? If you have one dollar and add fifty cents, thus increasing the amount by half, you have \$1.50, inflation notwithstanding. And now on to solving for unknowns.

## Solving For Unknowns

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It is correct to say that we have already been solving for unknowns in the previous examples. Frequently, we can see intuitively how to make a computation. It is more useful, however, to be able to systematically "set up" a problem and use mathematical processes; it is more efficient that way. So here goes. We'll look at the system, give some examples, and then we'll generalize.

The basic operation requires that we state the relationship of items by using symbols to represent the variable. The resulting statement is known as an equation. We have to have a balance, so the statement must show one group equal to another. Finding the unknown means that we must locate the value of one of the variables that will make this statement correct. Here we will be restricted to solving for only one variable, which requires knowledge of the value for every item in the relationship except one.

If we know that interest will be charged at 12 percent per annum on a loan of \$1,000, it is plain that the interest expense for using all the money for a whole year would be \$120.00. In this case, the unknown is the interest rate charge. (It does not take much strain to figure out its value.) But let's examine in more detail what we did.

In the dim distant past we learned this relationship: Simple interest expense is computed by multiplying the principal times the interest rate times the time for which the money is borrowed. To write a

statement showing this relationship for use in all situations, we need to assign symbols to each of the variables. The customary way to do this follows:

$$I = Prt$$

Where I is interest, P is principal, r is the interest rate in decimal form, and t is the time in annual terms. By substituting appropriate dollar, decimal, or fractional values for the symbols and using arithmetic, we can find the value we don't know. This will work here only if we know all but one of the variables. It doesn't matter which one we don't know; we can always find the correct value for it. There are some special rules for calculating simple interest, and we'll cover those a little later. Meanwhile, we can work with the equation as stated.

Note there are 4 variables in the equation. Let's assume data for three at a time, and systematically solve the equation for the missing information.

1. Find the interest charge for borrowing \$2,000 for 18 months at 12 percent per annum. (Here we know all but I.) So:

$$I = \$2000 * .12 * 3/2 = \$360$$

(We could have used 1.5 as the decimal equivalent of 3/2.)

Proof: interest each year would be \$240 and for half a year, \$120. Added together, the total interest is \$360. Proved.

2. Assume that the interest charge was given as \$400 to use \$2,000 for two years. What interest rate applied? (Here the unknown is r, and we know all the other values.)

In this case, we need to write an equation so that the unknown is on the left and everything we know is on the right. So:

$$r = I/Pt$$

This is found by taking the original equation and isolating the variable you want. The isolation is accomplished by dividing both sides by values that will cancel out to leave the unknown by itself. Here, in  $I = Prt$ , divide both sides by  $Pt$

$$I/pt = Prt/Pt$$

Note that on the right side, the P and t cancel out. (This is so because everything is being multiplied; it wouldn't work if we were adding or subtracting.) After canceling, we have

$$I/Pt = r$$

As stated before, we want the unknown variable to stand on the left, so we simply reverse the statement. Thus:



$$r = I/Pt$$

Now we're ready to substitute the information we have, and that will give us what we don't have.

$$r = \$400 / \$2000 * 2 = .10 \text{ Hence, } .10 * 100\% = 10\%$$

(Better find out who's lending funds at that rate!)

Proof: Interest at 10 percent a year would be \$200 on the \$2,000. For two years, the total would be \$400. Proved.

3. Assume that it would cost \$330 in interest to use a sum of money for 18 months. If the interest rate was 11 percent, how much is the loan? (Here we need to find the value for P.) So:

$$P = I/rt$$

(Divide both sides by  $rt$  to isolate  $P$ , then reverse the statement. For practice, write this out now.)

$$P = \frac{\$330}{.11 * 1\frac{1}{2}}$$

But notice we have a mixed number substituted for  $t$ . This could be changed to the decimal equivalent of 1.5 or the improper fraction. Thus:

$$P = \frac{\$330}{.11 * 1.5} = \frac{\$330}{.165} = \$2000$$

or

$$P = \frac{\$330}{.11 * 3/2} = \frac{\$330}{.33/2} = \frac{\$330}{.165} = \$2000$$

or

$$P = \frac{\$330 * 2}{.11 * 3} = \frac{\$660}{.33} = \$2000$$

Here there are three illustrations of the alternatives. In the last example, we followed the rule for dividing by a fraction: invert and multiply. All three procedures must produce the same answer.

Proof: If we borrow \$2,000 at 11 percent, the annual interest would be \$220; for a year and a half, the interest would be a total of \$330. Proved.

4. The interest charge is \$500, the amount borrowed is \$2,000, and the interest rate is 6 percent. Under those conditions, how long was the loan term? (Here the unknown is represented by  $t$ .) So:

$$t = \frac{I}{Pr}$$

(You decide how to obtain this restatement of the basic equation,  $I=Prt$ , and then prove that this is correct.)

$$t = \frac{\$500}{\$2,000 * .06} = \frac{\$500}{\$120} = 4.166 \text{ years}$$

Now that's an interesting number. How long is 4.166... years in English? Obviously, it is something over 4 years and less than 5. If we could change the .166... into a simple fraction, we could soon find the time in months, the common unit of measure. To convert this decimal to its fractional equivalent, we simply divide the decimal into 1. The result will be the denominator. Here  $1 / .16666... = 6$ . Therefore  $.16666... = 1/6$ . Now we can state that the time period (value for  $t$ ) is  $4 \frac{1}{6}$  years. The loan term then is 4 years and 2 months as  $1/6$  of a year, in months, is 2. ( $1/6$  of 12 = 2.)

*Proof: The interest on \$2,000 at 6 percent for one year is \$120. For 4 years, \$480 and for 2 months, \$20, for a total of \$500. Proved.*

A comment on the proof of each of these examples. Sometimes we prove a computation by the intuitive method as has been stated for each. The mathematical way is to go back to the original equation and substitute all of the values, including the one obtained by solving, to verify that the two sides of the equation are, in fact, equal. It cannot be overemphasized how important it is to confirm that your computation is correct before you go on to use the results.

You have been working with algebra. You may not have realized it, because we haven't been using  $x, y, z$ , etc. These symbols are always associated with algebra, aren't they? When you see an equation utilizing these symbols, recall that it simply states a relationship. Our very useful  $I = Prt$  can just as easily be written as:

$$x = Ayz$$

For example, as a custom, we designate the unknown variable by  $x$  and use other letters of the alphabet for constants and variables associated with what we need to know.

And now you know how to find an unknown. To make the most of this, you must remember the rules that apply to the various relationships we encounter in the mathematics of finance.

## Applications

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In the following material, detailed computational steps have not been included. In all cases, by following the system illustrated in the previous section, you should be able to confirm how the answers were

calculated. Here we will concentrate on information needed when creating the equation required to find an unknown.

## Interest

Interest is the charge or rent for the use of money. When funds are borrowed, this charge may be calculated only on a simple interest basis. Funds in a savings account earn compound interest.

To find the amount of simple interest, we use the formula or equation discussed before:  $I = Prt$ . The values  $I$  and  $P$  are always in dollars and cents. The interest rate must be expressed in decimal form for computing. A rate of 7 percent is written  $.07$ ; for 7 1/2 percent,  $.075$ , etc. It is customary to convert to a percentage for use in conveying information. Also it is considered always to be an annual rate unless indicated otherwise. Interest rates quoted for loans (or for savings accounts) are rates per annum. This means that the value substituted for the time variable,  $t$ , in the formula must also be in annual terms. We've seen how this might require a change from fractional to decimal equivalents. To emphasize, for six months,  $t$  would be written either as  $.50$  or  $1/2$ ; for 2 years and 3 months,  $t = 2.25$  or  $2\ 1/4$  or  $9/4$ .

Interest on a loan may be payable periodically, either with or without a payment on the principal. For many short-term loans, the interest is payable along with the principal at the end of the term. In that situation, we can solve for the payoff amount directly by using

$$S = P(1 + rt)$$

*where  $S$  is the sum due. Of course, we could also find  $S$  from  $S = P + I$ .*

Under the banker's rule, for short-term lending, the annual interest rate is converted to a monthly rate by dividing by 12. For example, 8 percent becomes  $2/3$  of 1 percent for each month, 6 percent is  $1/2$  percent per month, etc. This may be used to calculate the interest charge in dollars for one month. The total would be obtained by multiplying the number of months.

But what about compound interest? Think of a savings account. At the end of each interest period, the earned interest is added to the beginning balance to produce the new, larger balance for the next interest period. It is this conversion of interest to principal that prompts us to call the method compounding. In each time period after the first, interest is being earned on interest, assuming no withdrawal is made. Each interest computation, however, is made by using  $I = Prt$ . Compounding therefore involves simply a series of simple interest calculations. It must be observed that for each computation of interest, the value for  $P$  changes by the amount of earned interest added.

Now we must look at the frequency with which interest is converted. In the past, savings account interest was compounded every three months, or quarterly. Today, it is much more usual to find that interest is compounded every day, or daily. Some savings institutions use calculus and compound

savings interest continuously. It takes a fairly large deposit for the continuous compounding to produce more interest than when done daily. The more often interest is converted to principal, given the same annual rate, the greater will be the ending balance. When a saver must choose between differing interest rates and frequency of compounding, it can become a little difficult to determine which plan will earn the greatest income. Probably the simplest way to make this determination is to find out how much a \$100 deposit would grow to by the end of the first year under each of the alternatives. The largest ending balance will indicate the best plan. There may be early withdrawal penalties, minimum balance rules, and rate guarantees to consider as well.

There are, as you might expect, formulas to be used in finding unknowns when the relationship is that of compounding. The most frequently encountered unknown is the ending balance in a savings account given the principal, interest rate, and frequency of compounding. The formula to find  $S$ , the ending balance, is

$$S = P(1 + i)^n$$

*where  $i$  is the interest rate per compounding interval and  $n$  is the total number of such intervals for which  $S$  is being accumulated.*

In practice, it is unnecessary to use this formula because compound interest tables are readily available. They, of course, have been prepared by substituting various values for  $i$  and  $n$ . Let's take a look at how this would work if we needed to make a calculation (because we couldn't find the table). Assume that we deposit \$1,000 in a savings account that pays 5.25 percent compounded daily. (The advertised rate, 5.25 percent, is called the nominal rate.) What would the balance be at the end of the second year? Interest is left in the account, and no withdrawal or further deposits are made. From:

$$S = P(1 + i)^n$$

We get

$$S = \$1,000 * \left(1 + \frac{.052}{360}\right)^{720}$$

This would be rewritten as:

$$S = \$1,000 * (1.0001458)^{720}$$

To obtain the answer, all we'd need to do is multiply 1.001458 by itself 720 times! That would give us the total. Aren't you glad there are tables and computers?

Because  $i$  must be expressed as a rate per interest interval, we take the annual rate of 5.25 percent and divide it by 360 to get the daily interest rate. For simplicity, most financial institutions use 360 days in the year, but changes are occurring in this practice. It is likely that 365 will be universal before long. To find the value to use for  $n$  in the formula, we determine the total number of times interest is compounded. In a two year period, there would be  $2 \times 360 = 720$  in our example. This would be  $2 \times 365 = 730$  if the full year is used.

You may recognize the use of  $n$  here as a power. None of us has much trouble with squaring or even raising a value to the third power:  $5^2 = 25$ , or  $2^3$  (2 cubed) = 8, but raising it to the 720th power? There is no way this can be done by hand!

And just in case you wondered if the appreciation of value in property might be related to compounding, consider this example. Assume that a home is bought for \$80,000 and held for five years. If prices in the area rise at the rate of 5 percent a year, what would the prospective selling price be at the end of the holding period? One way to determine this would be to reason that at 5 percent each for 5 years, the total would be 25 percent. Hence the appreciated value would be \$100,000 (\$80,000 + 25% of \$80,000 = \$100,000.) Technically, if prices are increasing each year by 5 percent, the total increment would need to reflect compounding. Hence

$$S = P(1 + i)^n$$

would become :

$$\begin{aligned} S &= \$80,000(1.05)^5 \\ &= \$80,000(1.276275) = \$102,102 \end{aligned}$$

This example shows the difference compounding can make. Whether you can sell the property for \$102,102 or even \$100,000 five years hence is another matter.

As a final matter in compounding, for the purpose of comparison, it is customary to express the equivalent effective rate. This is defined as the rate to be applied once a year to produce the same result from using the nominal rate in calculating the rate used each compounding period. The effective rate can be used to make it easier to compare differing compounding plans.

In advertising for savings accounts you will see two interest rates quoted. The lower is the annual or nominal rate; the higher, the effective rate. If at the end of one year you applied, for example, 10.11% to the principal, the interest earned would be the same as if you applied 9.5% divided by 365 to the principal at the end of each day and then added the interest to produce the new, larger principal balance for the next day's calculation, repeating the process for the full year. And on the subject of names for interest rates, there are more.

## **Basis Points**

Perhaps you've seen the expression "basis points." This is used when discussing loan interest rates. One basis point is one one-hundredth of one percent. When interest rates increase from 12 percent to 13 percent, we can say the increase was 100 basis points. A change of half of one percent would then be termed fifty basis points. This is simply an alternate way of describing a measure.

# Discounting

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A discount is an allowance from a list price, for example. It certainly suggests a savings. When it comes to borrowing money, however, it means simply a reduction in the amount received.

A common practice in bank lending for personal and business loans is that of discounting the note. Stated simply, if you sign a note calling for repayment of \$5,000 at the end of one year, the lender may hand you \$4,300. The amount you receive is called the proceeds. The difference of \$700 is the discount. In effect, you are paying the interest in advance. That's all there is to it. On the due date you repay \$5,000. Your cost for using the money was \$700. The lender used a 14 percent discount rate (14 percent of \$5,000 for one year is \$700), but that is not an interest rate. Is the interest rate larger or smaller than 14 percent?

How about mortgage points? Assume that you have arranged a loan for \$70,000 for 30 years at 12 1/2 percent and 3 points. The lender would send \$67,900 to the escrow, yet you signed a note agreeing to repay \$70,000. The difference of \$2,100 (3 percent of \$70,000 is \$2,100) is a discount. Although you can treat this as interest expense for your personal income tax purposes if you use the property as your home, it is not really interest on your loan. The monthly payments (we'll cover these shortly) will be calculated on the basis of a beginning principal balance of \$70,000.

## Annual Percentage Rate (APR)

If you sign for \$70,000 and received only \$67,900, what rate of interest are you paying? This is where the annual percentage rate (APR), as specified by the Truth-in-Lending law, comes into play. The APR is simply the equivalent simple interest rate. That, recall, is the variable  $r$  in the equation  $I = Prt$  and  $r = I/Pt$ .

In the first discounting example above, where we signed for \$5,000 and received proceeds of \$4,300, we would calculate the APR, being  $r$ , by

$$r = \frac{\$700}{\$4,300} = .1628 = 16.28\%$$

*Note that the simple interest rate will always be larger than the equivalent interest rate (here it was 14 percent). Our interest payment of \$700 was made when we settled the debt by paying \$5,000 on the due date. During the term of loan, we had the full use of only \$4,300, not \$5,000, so the value of  $P$  is \$4,300.*

Now to the mortgage loan for which we had to pay points. We start out with only \$67,900 and have the use of it until we make the first payment. We know that we will have to make monthly payments of principal and interest. The interest portion of the monthly payment will be based on \$70,000, however.

It should be clear that we then are actually paying more than 12 1/2 percent, because we never did have our hands on the full \$70,000. This is why the question arises as to what the real simple interest rate is. How to calculate the APR here is not as simple as in the preceding example. Why? Because we make monthly repayments of the principal, so we are constantly reducing the loan balance P, unlike the case of the discounted \$5,000 loan. We could use the basic simple interest formula. First, we need to know the amount of the monthly payment and the portion charged to interest. For the time being, accept the amount as \$746.90; we'll discuss shortly how this is computed.

Assume that we receive the loan proceeds on the first of the month; the first payment of \$746.90 is due one month later. The interest owing would be:

$$I = \$70,000 * .125 * \frac{1}{12} = \$729.17$$

The balance of the payment, \$17.73 (\$746.90 - \$729.17 = \$17.73) will be credited to the principal. So:

$$r = \frac{\$729.17}{\$67,900 * \frac{1}{12}} = .1289 = 12.89\%$$

the approximate APR, which is greater than 12 1/2 percent as we expected. The value for P here is the \$67,900, the actual funds borrowed, not the \$70,000 face of the note.

For a more accurate calculation of the APR, annuity formulas are used. We will see something of these in the next section. Meanwhile, you might recall an earlier rule of thumb offered for estimating the APR when points are charged. We said that each point is worth about 1/8 of 1 percent. Here, then, we'd have an approximate APR of 12 1/2 + (3 x 1/8) = 12 7/8 %. But there is more to discounting.

## Discounting Second Mortgages

Earlier we described the nature and risks involved in lending secured by junior liens. A seller who reluctantly takes a second mortgage because the buyer doesn't have enough cash may be looking for a way out after the deal is closed. He may be willing to sell the mortgage at a discount. Because the risk often is greater on seconds compared to the first mortgage, the buyer of these mortgage instruments can command a substantial reduction.

Discounts of 30 percent, 40 percent, and even higher are not uncommon. To illustrate, let's assume that the original second deed of trust was in the amount of \$5,000. A trust deed buyer may offer a 40 percent discount. If this is accepted, the price paid would be:

$$\$5,000 \text{ less } 40\% \text{ of } \$5,000 = \$3,000$$

*(Alternatively, you could simply take 60 percent of the note balance.)*

After the note changed hands, the original debtor would be advised to make his payments to the new holder. Note that this sale does not affect the second mortgage. His payment amounts, interest rate, and other terms remain the same. One question that may arise is: What rate is the trust deed purchaser earning on his investment?

The easiest way to calculate an approximate yield would be to use our old friend

$$r = \frac{I}{Pt}$$

If the note was originally written at 10 percent with payments of ,say, \$50 per month, the interest component of the first payment would be 10 percent of \$5,000/12 = \$41.67. So:

$$r = \frac{\$41.67}{\$3,000 * \frac{1}{12}} = 16\frac{2}{3}\% \text{ approximately.}$$

How about that for an investment yield? (Don't forget the risk) But there's more to the yield. If the debt is ultimately paid off, the holder will receive a total of \$5,000 in principal payments, which is \$2,000 more than he paid for the note. To compute the precise yield, it would be necessary to use annuity formulas, and that is beyond the scope of this material. In this example, the ultimate yield would be substantially more than 16 2/3 percent per annum.

Another item that may enter into yields is a prepayment penalty. If the note contains a provision for this penalty, the holder of the note will receive more than the unpaid balance at a premature payoff. that bonus would serve to increase the percentage yield.



## Chapter 2 Review Questions – Section 1

1. Tom sells his house through a broker to whom he pays a 7% commission. If Tom nets \$6,500 after paying the broker, an existing mortgage of \$23,400, and a property tax lien of \$512, what was the selling price of the house?

- A. \$23,912
- B. \$30,412
- C. \$32,701
- D. \$23,400

2. Greg borrowed \$10,000 to buy a piece of land and agrees to pay the bank \$95 per month, applied first to interest and then to principal. If the interest rate is 8.5%, what will the outstanding balance be at the end of the first month?

- A. \$9,905.00
- B. \$9,929.17
- C. \$9,975.83
- D. \$10,755.00

3. Mr. Wimple sold his house for \$48,500, which was 13% more than he paid for it. How much did he pay for the house?

- A. \$48,500
- B. \$42,920
- C. \$54,805
- D. \$55,747

4. David sells a piece of land for \$24,450. If he netted \$22,311 after paying a broker's commission, the rate of commission paid was 9.6%. True or False?

5. A five-year-old house has appreciated 6% annually each year. If the house was worth \$66,500 when it was built, what is its value today?

- A. \$46,550
- B. \$66,500
- C. \$83,955
- D. \$88,992

6. An insurance company pays 90% of any fire damage over the \$200 deductible. How much will an owner receive if he suffers an \$835 fire damage?

- A. \$571.50
- B. \$635.00
- C. \$180.00
- D. \$751.50

# Amortization

---

When we pay off a mortgage by making periodic payments of principal and interest, it is said that we are amortizing the debt. The basic calculation required is that of the amount of the periodic payment. Here's a new term : annuity. A series of equal payments, made or received, at equal time periods, is known as an annuity. Hence mortgage payments form an annuity. This fact allows us to use some very useful mathematical tools associated with annuities.

As we think of the series of monthly payments in the preceding example, we can see that each month a portion of the payment goes to meet the interest charge, and the balance is applied to the principal. It is customary for the monthly payment to remain constant; thus the components change each month. Interest is charged against the payment first so as it reduces, the amount available to apply to the principal increases. (If you are saying to yourself, "my mortgage payments don't remain the same." you are forgetting that if your payment includes funds for the loan trust fund or impound account for taxes and insurance, it is that part of the payment that is subject to change; the principal and interest combined amount is constant throughout the loan payoff period.)

The following excerpt from an amortization schedule demonstrates the allocation of principal and interest for the first year.

Note that the interest expense for the year plus the amount by which the loan is reduced (equity build) equals the total of the monthly payments. This is a very useful relationship to keep in mind. Here we have  $\$1790.95 + \$223.13 = \$2014.08$ .

For amortization, we need a monthly payment amount that is exactly enough to satisfy the interest requirement and produce enough to pay off the principal balance. The final payment - here the one at the end of 30 years (the 360th) - would be exactly enough to cover the interest due, and the remainder would extinguish the debt completely. So how would you make such a calculation?

## AMORTIZATION SCHEDULE

Loan Amount: \$20,000; Interest Rate: 9%; Mortgage  
Term: 25 years; Monthly Payment: \$167.84

<i>Number</i>	<i>Unpaid Principal</i>	<i>Interest</i>	<i>Balance</i>
1	17.84	150.00	19982.2
2	17.97	149.87	19964.2
3	18.11	149.73	19946.1
4	18.24	149.60	19927.8
5	18.38	149.46	19909.5
6	18.52	149.32	19890.9

7	18.66	149.18	19872.3
8	18.80	149.04	19853.5
9	18.94	148.90	19834.5
10	19.08	148.76	19815.5
11	19.22	148.62	19796.2
12	19.37	148.47	19776.9
	<u>223.13</u>	<u>1790.95</u>	

We'll avoid examining how the annuity formulas are prepared. Instead, just to get the flavor of the computational process, we'll look at how the monthly payment for the illustration would be determined. The basic formula here would be:

$$R = A \frac{i}{1 - (1+i)^{-n}}$$

where R is the periodic payment, A is the original mortgage balance, i is the interest rate according to the frequency of the periodic payment, and n is the total number of payments to be made. In our case, i would be  $.1250/12 = .0105$ . The value of n is  $30 \times 12 = 360$ .

$$R = \$70,000 \frac{.0105}{1 - (1.0105)^{-360}} = \$746.90$$

If you obtain a slightly different number from a table, it will be due to the difference in rounding during the computation. You probably feel that it would be easier just to use a table whenever you needed to know how much the mortgage payment would be. Appendix A should be useful; it provides a table of loan constants.

Back to the APR. As stated, the most accurate value will be determined by using the annuity formulas. All that is needed is the value for i to satisfy the formula when the appropriate values for the other variables have been substituted. In our example, we would need to solve for i in

$$\$746.90 = \$67,900 \frac{i}{1 - (1+i)^{-360}}$$

How this would be done is beyond the scope of this book. Perhaps you are willing to take on faith that it can, in fact, be done. Once the value of i is computed, it then would be multiplied by the number of times the payment is made each year; here that would be 12. The result would be converted to a percentage by multiplying by 100 percent. In this case, it would be close to 12.88 percent.

As covered elsewhere, the APR is computed by considering several items, including points. In practice, a lending officer would take the total of the items prescribed by the Truth-in-Lending law and divide it into

the loan amount. The result is a factor to be used with a table of APRs related to the factor, the interest rate, and the mortgage term.

### Depreciation (For Tax Purposes)

Depreciation (for income taxes) is a diminishing (loss) in the value of buildings and other improvements. All new depreciation schedules for normal income tax purposes involving real property must be straight-line. *Straight-line depreciation is a method of computing depreciation on assets other than land for income tax purposes in which the difference between the original cost and salvage value is deducted in installments evenly over the life of the asset.* It is based upon the assumption that depreciation is dependent solely on the passage of time. Depreciation is spread uniformly over the useful life of a property (building). *Note:* When doing depreciation problems, it is important to remember that land does NOT depreciate.

To find depreciation using the straight-line method use:

$$\text{Annual depreciation} = \frac{\text{Value(cost)of improvements}}{\text{Economiclife}}$$

Example: Mr. and Mrs. Roberts purchased some real property for \$475,000. The land was valued at \$200,000. The improvements had an estimated economic life of 27.5 years. What would be the depreciated value of the property after 17 years? To answer, we must first determine the value of the depreciable asset (improvements) because land is not depreciated.

\$475,000 (land and improvements)

- 200,000 (land)

\$275,000 (Cost of improvements)

$$\text{Annual depreciation} = \frac{\$275,000}{27.5} = \$10,000 \text{ depreciation per year}$$

\$10,000 x 17 years = \$170,000 accumulated depreciation

\$275,000 - \$170,000 = \$105,000 depreciated value of the improvements only

\$105,000 + \$200,000 (value of land) = \$305,000

\$305,000 is the depreciated value of the property

*Note: The IRS allows straight line depreciation of 27.5 years on residential properties and 39 years on commercial properties.*

## Stipulated Amount Payments

---

In the foregoing, we found how to compute the exact monthly payment needed to pay the interest and repay the principal over the mortgage term. Especially in second mortgages, we encounter the balloon payment and a monthly payment that does not serve to amortize the debt over a specified term. As a rule, the monthly payment for a junior lien is an arbitrary amount, such as 1 or 1 1/2 percent of the original loan. From each payment is first deducted interest to date, and the balance is applied to the principal; the new reduced principal is used in calculating the next interest charge. Several questions may arise: What will the mortgage balance be at the end of, say, 5 years, such as on a due date? Just how long would it take for the arbitrary payment to pay off the debt if there wasn't a due date? Those questions make it necessary for us to take a closer look at this situation.

Recall the typical circumstance. The seller agrees to take back a second mortgage for \$20,000 at 14 percent interest and a due date 5 years away. The monthly payment is to be \$300. It is useful to be able to determine in advance what the unpaid principal balance will be on the due date. This is a balloon payment. An alternative to the arbitrary percent method for determining the monthly payment is to use, instead, the payment that would amortize the debt, say, in 15 years, even though there is a five-year due date.

As discussed in the amortization section, there are tables available to use in determining unpaid principal balances when the monthly payment applies for the mortgage term. These tables would not apply if the monthly payment was computed by applying an arbitrary percentage, such as percent, to the loan amount. They would apply if the payment was for a given term, even though a due date serves to declare the debt balance payable before the end of the term.

Some inexperienced private lenders may stipulate a monthly payment that is inadequate to even cover the interest, let alone serve to reduce the principal. If a payment amount is arbitrarily chosen as a percentage of the initial loan balance, it should never be less than the mortgage interest rate divided by 12. If it is, the payment will not be enough to cover the interest. If it is equal to this value, the payment will be equal to the interest, so nothing will be available to apply to the principal.

Back now to our \$300 second mortgage payment. At the end of the first month, the payment would be apportioned as follows:

Interest	\$233.33
Principal	<u>66.67</u>
	\$300.00

For the second month:

Interest	\$232.56
Principal	<u>67.44</u>
	\$300.00

The borrower will want to know how big the balloon payment will be on the due date. As can be seen, the unpaid second mortgage balance could be calculated by repeating the process shown until the due date is reached, which is an obviously undesirable task. Instead, by using tables prepared for such a purpose, it becomes a relatively simple proposition. These tables and the formulas are beyond the scope of this text and are not included. It is emphasized that care must be taken not to use an amortization table if the monthly payment has been arbitrarily determined as shown here. Instead, look for a balloon payment or stipulated payment table.

## Mortgage Payoff

---

What about calculating the amount needed to pay off the mortgage? Lenders, of course, maintain records and provide borrowers either with a loan payment record book or a computer printed statement at the end of each year. The loan balance applies to the beginning of a payment period; interest is always charged in arrears. Assume that the balance outstanding as of July 1 (a payment date) is \$24,500 and the annual interest rate is 9 percent. How much would it take to pay off the mortgage on July 10? There is more than one way to make the calculation. Here is one of them.

Interest for one month is computed:

$$\frac{3}{4} \text{ of } 1\% \text{ of } \$24,500$$
$$.0075 * \$24,500 = \$183.75$$

*(For convenience each month is considered to have 30 days)*

The interest for 10 days would be :

$$\frac{1}{3} \text{ of } \$183.75 = \$61.25$$

So the payoff amount would be :

$$\$24,500 + \$61.25 = \$24,561.25$$

## Add-On Interest

---

The method used for charging interest and calculating monthly payments for mortgages involves computing the charge on the unpaid principal balance. When loans are made for other purposes, lenders may use instead the add-on interest method. This is often encountered in automobile and other personal loan financing and in making loans on mobile homes. Because real estate practitioners can now handle used mobile home sales, it is important that this financing method be understood.

Assume that the lender has decided its add-on financing rate is 9 percent per year. For a \$15,000 loan to be repaid over 10 years, the monthly payment would be computed as follows:

Loan	\$15,000
Add-on interest	<u>13,500</u>
	\$28,500
	$\$28,500 / 120 = \$237.50$

The interest charge amounts to 9 percent of the loan for each year. Here we have  $.09 \times \$15,000 = \$1,350 \times 10 = \$13,500$ . There are 120 monthly payments to make in 10 years.

What if the loan is paid off in advance? The interest is determined by using the "rule of 78s." This is an adaptation of the sum-of-the-years digits method used in calculating depreciation. In order to recover the expense incurred in making the loan, the lender allocated a greater proportion of interest over the loan term. This is reflected in the amount of the unpaid balance due (payoff amount) if the payments are not made for the entire loan period. While this has been a customary practice for many years, some lenders are discarding the add-on method, so it is no longer universal. Some financial institutions now simply charge interest on the unpaid principal balance for all installment payment loans.

Let's assume that the loan has run for 4 years. What is the unpaid balance due at the time under the rule of 78s? The first step requires that we determine the total paid to date.

$$\$237.50 * 48 = \$11,400$$

Now for the unearned interest (finance) charge.

---

1. The sum of the total number of months is 7,260. (add the digits from 1 to 120.)

---



2. The sum of the number of months remaining is 2,628. (Add the digits from 1 to 72; there are six years remaining.)

3. Apply the fraction formed by	2,628	
these two numbers against the	-----	x \$13,500 = \$4,887
original financing charge:	7,260	

4. The payoff amount:	Original Loan		\$28,500
	Less:		
	Paid	\$11,400	
	Unearned interest	4,887	<u>16,287</u>
	Balance due		\$12,213

The add-on method will produce a greater charge for interest than if interest at the same rate is charged on the unpaid principal balance, as is customary for home mortgages.

Lenders using the add-on interest method must give borrowers the equivalent simple interest rate, the APR. Tables are available for that calculation. As a general rule, the APR will be nearly twice the add-on interest rate. Here, with a 9 percent add-on rate, the APR would approach 17 percent.

## Income Capitalization

In real estate investments, you will encounter several methods for computing maximum economic value of income producing property. One of those methods requires the computation of a yield expressing the income as a return on the investment. The concept will be treated in more detail there. Meanwhile, we can look at how mathematical tools are employed.

Assume that the offering price of an apartment is \$300,000. The annual gross rental income is \$36,000, and the operating expenses total \$12,000 each year. We disregard mortgage payments, interest expense, depreciation, and income tax considerations. On a cash basis, we take in \$36,000, pay out \$12,000, and have \$24,000 net income remaining as a return on the investment of \$300,000. The relationship may be written as:

$$\text{Yield} = \frac{\text{net income}}{\text{purchase price}} * 100\%$$

Using the processes discussed so far, if we know any two of these, we can find the third.

1. The yield is:

$$\frac{\$24,000}{\$300,000} * 100\% = 8\% \text{ per year}$$

2. Assume that you wanted a 10 percent yield. What is the most you should pay for a property yielding a net annual income of \$24,000?

$$\text{Purchase Price} = \frac{\text{net income}}{\text{yield}} = \frac{\$24,000}{.1} = \$240,000$$

Technically, the term capitalization of income applies only to example 2.

3. Assume that you wanted a 10 percent yield. If the offering price was \$300,000, the net annual income should be not less than what amount?

$$\text{Net income} = \text{yield} \times \text{purchase price}$$

$$\text{Net income} = .10 \times \$300,000 = \$30,000$$

For review, compare how this basic equation for three variables has been used in the same way we worked with  $I = Prt$ , an equation with 4 variables.

## Area Measurement

---

The measurement of the distance from one point to another is called linear measurement. Usually this is along a straight line, but it can also be along a curved line. Distance is measured in inches, feet, yards and miles. Less commonly used are chains (66 feet) and rods (16 and a half feet). Surface areas are measured in square feet, square yards, acres (43,560 square feet) and square miles. In the metric system, the standard unit of linear measurement is the meter (39.37 inches). Land area is measured in square meters and hectares. A hectare contains 10,000 square meters or 2,471 acres.

To determine the area of a square or rectangle, multiply its length times its width. The formula is:

$$\text{Area} = \text{Length} \times \text{Width}$$

$$A = L \times W$$

### EXAMPLE:

A parcel of land measures 660 feet by 330 feet. How many square feet is this?

$$\text{Area} = 660 \text{ feet} \times 330 \text{ feet} = 217,800 \text{ square feet}$$

How many acres does this parcel contain?

$$\text{Acres} = 217,800 / 43,560 = 5 \text{ acres}$$

If a buyer offers \$42,500 for this parcel, how much is the offering per acre?

$$\$42,500 / 5 = \$8,500$$

To determine the area of a right triangle, multiply one-half of the base times the height:

$$A = \frac{1}{2} * B * H$$

$$A = \frac{1}{2} * 25 * 50$$

$$A = 625 \text{ sq ft}$$

To determine the area of a circle, multiply 3.14 times the square of the radius:

$$\text{Area of Circle} = 3.14 * r^2$$

*Note: Where the diameter of a circle is given, divide by two to get the radius.*

To determine the area of composite figures, separate them into their various components. Thus:

## Volume Measurement

Volume is measured in cubic units. The formula is:

$$\text{Volume} = \text{Length} * \text{Width} * \text{Height}$$

$$V = L * W * H$$

For example, what is the volume of a room that is 10 ft by 15 ft with an 8 ft ceiling?

$$V = 10' * 15' * 8'$$

$$V = 1,200 \text{ cu ft}$$

**CAUTION:** When solving area and volume problems, make certain that all the units are the same. For example, if a parcel of land is one-half mile long and 200 ft wide, convert one measurement so that both are expressed in the same unit; thus the answer will be either in square feet or in square miles. There is no such area measurement as a mile-foot. If a building is 100 yards long by 100 feet wide by 16'6" high, convert to 300 ft by 100 ft by 16.5 before multiplying.

If the label on a five-gallon can of paint says it will cover 2,000 square feet, how many gallons are necessary to cover 3,600 sq ft?

A problem like this can be solved two ways:

One way is to find out what area one gallon will cover. In this case  $2,000 \text{ sq ft} / 5 \text{ gallons} = 400 \text{ sq ft per gallon}$ . Then divide 400 sq ft/gal into 3,600 sq ft and the result is 9 gallons.

The other method is to set up a proportion:

This reads, "5 gallons is to 2,000 sq ft as 'Y' gallons is to 3,600 sq ft." To solve for "Y", multiply both sides of the proportion by 3,600 sq ft. Thus:

Divide 2,000 sq ft into 3,600 sq ft and multiply the result by 5 gallons to get the answer.

## Front-Foot Calculations

When land is sold on a front-foot basis, the price is the number of feet fronting on the street times the price per front foot.

$$\text{Price} = \text{front footage} \times \text{rate per front foot}$$

Thus a 50 ft x 150 ft lot priced at \$1,000 per front foot would sell for \$50,000. Note that in giving the dimensions of a lot, the first dimension given is the street frontage. The second dimension is the depth of the lot.

## Chapter 2 Review Questions – Section 2

7. A lot measures 411.4' x 900'. Its total worth is \$48,000 if land is valued at \$6,000 per acre. True or False?
8. What is the rate of return on an investment when net operating income is \$3,430 and the property value is \$22,000?
- A. 1.56%
  - B. 641%
  - C. 15.6%
  - D. .156%
9. The gross rent multiplier for a house that has a market value of \$55,000 and rents for \$500 per month is 100. True or False?
10. An agent sells homes on an 8% commission. What is the total sales value of properties sold for her to receive \$7,500 in commission?
- A. \$100,000
  - B. \$125,000
  - C. \$75,000
  - D. \$93,750
11. The compound interest on \$40,000 at 14% compounded quarterly for 1 year is \$5,600. True or False?
12. Find the interest on \$5,400 at 3% for 90 days?
- A. \$40.50
  - B. \$162.00
  - C. \$50.40
  - D. \$0.45

13. Last year a family paid \$33,600 for house payments. If this is equal to 30% of the family's yearly income, the yearly income was \$33,000. True or False?

14. A farmer sold  $62\frac{1}{2}$  acres of a 750 acre farm. Express as a fraction what part of the farm was sold?

- A.  $\frac{1}{16}$
- B.  $\frac{1}{12}$
- C.  $\frac{1}{3}$
- D.  $\frac{1}{8}$

15. Ann and Mary invested \$86,000 in purchasing a home. Mary invested \$9,000 more than Ann. How much did Ann invest?

- A. \$38,500
- B. \$77,000
- C. \$86,000
- D. \$48,000

16. What is the yield (annual return on the investment) for a house that has a market value of \$60,000 and rents for \$500 per month?

- A. 10
- B. 0.10
- C. 0.0083
- D. 120

# Glossary

**Accounting:** the orderly arrangement, display, and analysis of business transactions.

**Accrual Basis Accounting:** a system in which allowance is made in the records for noncash items, such as depreciation, and for apportioning income or expenses over time.

**Annual Percentage Rate (APR):** as specified by the Truth-in-Lending law, the equivalent simple interest rate.

**Annuity:** a series of equal payments, made or received, at equal time periods.

**Balance Sheet:** financial statement reporting, at a given date, the total amount of assets held by a firm and the liabilities and owners' equity that finance these assets.

**Basis Point:** one one-hundredth of one percent.

**Bookkeeping:** the processes and procedures used to record measures of dollars and cents, according to categories.

**Cash Basis Accounting:** the process of keeping the most simple set of records, containing only information concerning actual cash coming in and going out.

**Depreciation:** a diminishing (loss) in the value of buildings and other improvements. There are several accounting methods that are used in order to write off an asset's depreciation, including straight-line depreciation.

**Discount:** an allowance from a list price.

**Double-Entry System:** a system to indicate that for each transaction, a record has been made in at least two places.

**Fraction:** simply a number, representing a quantity that is smaller than one or a whole, and has two parts: a numerator and a denominator.

**Income Statement:** financial statement displaying the totals of income and expenses by categories of transactions.

**Interest:** the charge or rent for the use of money.

**Linear Measurement:** the measurement of the distance from one point to another.

**Yield:** expressing the income as a return on the investment.

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# Review Question Answers

## Chapter 1 Review Questions – Section 1

1. The basic financial statements include the balance sheet, income statement, statement of cash flows, statement of retained earnings, and statement of changes in retained earnings. True or False?

True is incorrect. The basic required statements are the balance sheet, income statement, and statement of cash flows.

False is correct. The basic statements do not include a statement of retained earnings and statement of changes in retained earnings.

2. The primary purpose of the balance sheet is to reflect

- A. Incorrect. The measurement attributes of assets and liabilities include but are not limited to fair value.
- B. Incorrect. Financial statements reflect the going concern assumption. Hence, they usually do not report forced liquidation values.
- C. **Correct.** The balance sheet presents three major financial accounting elements: assets, liabilities, and equity. Assets are probable future economic benefits resulting from past transactions or events. Liabilities are probable future sacrifices of economic benefits arising from present obligations as a result of past transactions or events. Equity is the residual interest in the assets after deduction of liabilities.
- D. Incorrect. The future value of a company's stock is more dependent upon future operations and investors' expectations than on the data found in the balance sheet.

3. Bookkeeping is:

- A. **Correct.** The processes and procedures used to record measures of dollars and cents, according to categories, are called bookkeeping.
- B. Incorrect. The orderly arrangement, display, and analysis of these numbers are termed accounting.
- C. Incorrect. Bookkeeping does not involve financial analysis.
- D. Incorrect. Compliance with financial accounting standards is an area in financial accounting and reporting.

4. Balance sheet is also called:

- A. Incorrect. An income statement measures operating performance of an entity for a specified period of time.
- B. **Correct.** A balance sheet tells you about the financial position of an entity and is therefore called a statement of financial position.
- C. Incorrect. A financial analysis statement involves the analysis of financial statements.
- D. Incorrect. A profit or loss statement is an income statement.

5. Business managers want to know the operating results of financial transactions occurring during a specific time period as well as business' overall position at any instant in time. True or False?

True is correct. While we may want to know the operating performance of financial transactions occurring during a specific time period, we also need to know our overall position at any instant in time. For that we have a balance sheet. Taken together, an income statement and a balance sheet will constitute a set of financial statements.

False is incorrect. To evaluate a business' overall financial fitness, we need know about its operating performance for a specified time period and financial position at a particular point in time.

6. The term "double-entry system" refers to

- A. Incorrect. The distinction between real and nominal accounts is based on the relative permanence of accounts rather than the double entry, self-balancing attribute of accounting systems.
- B. **Correct.** In the double-entry system, each transaction is composed of two parts, debits and credits. The debits must equal the credits, and the sum of all debits for all transactions in a double entry system must equal the sum of all credits.
- C. Incorrect. Many journals may be used, e.g., general journal, sales journal, cash receipts journal, and cash payments journal.
- D. Incorrect. Even though journals and ledgers are parts of all double entry systems, they have nothing to do with the term "double entry."

7. A chart of accounts is

- A. Incorrect. Actual transactions are not flowcharted. Flowcharts of accounting procedures are developed by auditors and systems analysts (but are not called charts of accounts).

- B. Incorrect. An accounting procedures manual explains how to use the chart of accounts, e.g., whether to make adjusting entries, reversing entries, etc.
- C. Incorrect. A journal contains the initial recording of the transactions that affect the accounts contained in the chart of accounts.
- D. **Correct.** A chart of accounts is a listing of all account titles used within an accounting system. Business transactions affecting these accounts are initially recorded by journal entries and then posted to the individual accounts maintained in the ledger.

## Chapter 1 Review Questions – Section 2

8. Accrual basis accounting only involves information about actual cash coming in and going out. True or False?

True is incorrect. When allowance is made in the records for noncash items, such as depreciation, and for apportioning income or expenses over time, we designate the method as *accrual basis accounting*.

False is correct. Cash basis accounting involves information concerning actual cash coming in and going out.

9. The best indication of an enterprise's present and continuing ability to generate favorable cash flows is information about enterprise earnings based on accrual basis of accounting. True or False?

True is correct. Information about enterprise earnings based on accrual accounting generally provides a better indication of the enterprise's present and continuing ability to generate favorable cash flows than would information limited to the financial effects of cash receipts and payments. Accrual accounting attempts to record the financial effects on an enterprise of transactions and other events and circumstances that have cash consequences in the periods in which those transactions, events, and circumstances occur; rather than only in the periods in which cash is received or paid by the enterprise.

False is incorrect. Cash basis accounting does not match revenues earned with the expenses incurred to earn those revenues.

10. What function do general ledgers serve in the accounting process?

- A. Incorrect. Accounting data are reported in the financial statements.
- B. Incorrect. Data are summarized during the adjusting and closing process.
- C. **Correct.** General ledgers serve to classify accounting data. Transactions that have been recorded in the journals are posted to the general ledger accounts where they are classified as to the accounts that have been affected.
- D. Incorrect. Transactions are recorded in the journals.

11. Financial statements prepared by non-CPAs are:

- A. Incorrect. When a CPA has prepared a financial statement, a certification will appear on it to the effect that it conforms to generally accepted accounting principles. That does not mean that statements prepared by non-CPAs are unsatisfactory.
- B. **Correct.** When a CPA has prepared a financial statement, a certification will appear on it to the effect that it conforms to generally accepted accounting principles. That does not mean that statements prepared by non-CPAs are unsatisfactory. A noncertified statement, however, can be just as accurate.
- C. Incorrect. Financial statements prepared by non-CPAs are not certified.
- D. Incorrect. A noncertified statement can be just as accurate.

12. Revenue deductions do NOT include:

- A. Incorrect. Revenue deductions include state income tax withheld.
- B. **Correct.** Income tax refund is a revenue addition. Revenue deductions include state income tax withheld, state disability insurance, taxes, health insurance premium, life insurance, and car insurance.
- C. Incorrect. Revenue deductions include health insurance premium.
- D. Incorrect. State disability insurance taxes are included in revenue deductions.

13. The balance sheet assets do NOT include:

- A. Incorrect. Cash on hand is an asset account of the balance sheet.
- B. Incorrect. It is an asset account.
- C. Incorrect. Real estate is an asset account.
- D. **Correct.** Accounts payable is a liability account.

14. Which of the following statement is incorrect?

- A. Incorrect. A current asset is one that is in the form of cash or can be expected to be turned into cash during the next year.
- B. Incorrect. A current liability is an obligation payable within the next 12 months.
- C. Incorrect. The solvency of a business is the difference between the current assets and the current liabilities.
- D. **Correct.** The current assets should be at least twice the current liabilities. In the event a financial crisis develops, a business should be able to pay its current obligations out of its current assets and have some to spare. If the current liabilities exceed the current assets, the firm is in financial difficulty.

## Chapter 2 Review Questions – Section 1

1. Tom sells his house through a broker to whom he pays a 7% commission. If Tom nets \$6,500 after paying the broker, an existing mortgage of \$23,400, and a property tax lien of \$512, what was the selling price of the house?

- A. Incorrect. "\$23,912" is the sum of an existing mortgage and taxes.
- B. Incorrect. This is just the sum of a net proceed of \$6,500, an existing mortgage of \$23,400, and a property tax lien of \$512.
- C. **Correct.** The equation is:  $(x - 0.07x - \$23,400 - \$512) = \$6,500$ , where  $x$  is the selling price.  $0.93x = \$30,412$ . Solving for  $x$  yields \$32,701.
- D. Incorrect. "\$23,400" is an existing mortgage number.

2. Greg borrowed \$10,000 to buy a piece of land and agrees to pay the bank \$95 per month, applied first to interest and then to principal. If the interest rate is 8.5%, what will the outstanding balance be at the end of the first month?

- A. Incorrect. It is the difference between \$10,000 and \$95.
- B. Incorrect. It is the difference between \$10,000 and \$24.17, just the interest.
- C. **Correct.** The interest is  $\$10,000 \times .085/12 = \$70.83$ . The monthly payment of \$95 per month allows the principal to be reduced by \$24.17 ( $\$95 - \$70.83$ ). So the outstanding balance at the end of the first month is \$9,975.83 ( $\$10,000 - \$24.17$ ).
- D. Incorrect. "\$10,755.00" is a negative amortization figure.

3. Mr. Wimple sold his house for \$48,500, which was 13% more than he paid for it. How much did he pay for the house?

- A. Incorrect. This is his home price.
- B. **Correct.**  $x + .13x = \$48,500$ ;  $1.13x = \$48,500$ . Solving for  $x$  yields \$42,920.

- C. Incorrect. "\$54,805" is a 13% increase in his home price.
- D. Incorrect. This is number derived from dividing \$48,500 by .87.

4. David sells a piece of land for \$24,450. If he netted \$22,311 after paying a broker's commission, the rate of commission paid was 9.6%. True or False?

True is incorrect.  $(\$24,450 - \$22,311)/\$24,450$ ;  $\$2,139/\$24,450=8.75\%$

False is correct. 9.6% is the number derived from  $(\$24,450 - \$22,311)/\$22,311 = \$2,139/\$22,311$ .

5. A five-year-old house has appreciated 6% annually each year. If the house was worth \$66,500 when it was built, what is its value today?

- A. Incorrect. This is amount depreciated for 5 years.
- B. Incorrect. This is the house's current worth.
- C. Incorrect. This is the amount worth from 4 years from now.
- D. **Correct.**  $\$66,500(1 + .06)^5 = \$88,922$

6. An insurance company pays 90% of any fire damage over the \$200 deductible. How much will an owner receive if he suffers an \$835 fire damage?

- A. **Correct.**  $(\$835 - \$200) \times 90\% = \$571.50$ .
- B. Incorrect. This is the amount of \$835 fire damage minus the \$200 deductible.
- C. Incorrect. This is 90% of \$200 (deductible).
- D. Incorrect. This is 90% of \$875 (fire damage).

## Chapter 2 Review Questions – Section 2

7. A lot measures 411.4' x 900'. Its total worth is \$48,000 if land is valued at \$6,000 per acre. True or False?

True is incorrect. A lot is 370,260 sq. ft. One acre is 43,560, so the lot is 8.5 acres. Thus, 8.5 acres x \$6,000 = \$51,000.

False is correct. \$48,000 is the value for 8 acres.

8. What is the rate of return on an investment when net operating income is \$3,430 and the property value is \$22,000?

- A. Incorrect. The decimal is misplaced. The correct return should be  $\$3,430/\$22,000 = 15.6\%$
- B. Incorrect.  $(\$22,000/\$3,430) \times 100 = 641$ . The return should be  $(\$3,430/\$22,000) \times 100 = 15.6\%$
- C. **Correct.**  $\$3,430/\$22,000 = 15.6\%$
- D. Incorrect. The correct return is  $\$3,430/\$22,000 = 15.6\%$ . The decimal is misplaced.

9. The gross rent multiplier for a house that has a market value of \$55,000 and rents for \$500 per month is 100. True or False?

True is incorrect. This is the case where the rent is \$550;  $\$55,000/\$550 = 100$ .

False is correct. The correct multiplier is  $\$55,000/\$500 = 110$ .

10. An agent sells homes on an 8% commission. What is the total sales value of properties sold for her to receive \$7,500 in commission?

- A. Incorrect. This is the case for a 7.5% commission;  $\$100,000 (= \$7,500/.075)$ .
- B. Incorrect. . This is the case for a 5% commission;  $\$150,000 (= \$7,500/.05)$ .
- C. Incorrect. This is the case for a 10% commission;  $\$75,000 (= \$7,500/.1)$ .
- D. **Correct.**  $.08x = \$7,500$ . Solving for x yields \$93,750.

11. The compound interest on \$40,000 at 14% compounded quarterly for 1 year is \$5,600. True or False?

True is incorrect. The quarterly interest rate is 3.5% ( $14\%/4$ ). The compound amount at the end of the year is  $\$40,000(1 + .035)^4 = \$45,900$ . So the total is \$5,900 ( $\$45,900 - \$40,000$ ).

False is correct. This is the yearly compounded amount;  $\$40,000 \times 14\% = \$5,600$ .

12. Find the interest on \$5,400 at 3% for 90 days?

- A. **Correct.**  $\$5,400 \times (.03/360) \times 90 \text{ days} = \$40.50$
- B. Incorrect. This is the yearly interest;  $\$5,400 \times 3\%$ .

- C. Incorrect. The number was transposed. The correct amount is  $\$5,400 \times (.03/360) \times 90 \text{ days} = \$40.50$
- D. Incorrect. This is a daily interest;  $\$5,400 \times (.03/360) = \$0.45$ . The correct amount is  $\$5,400 \times (.03/360) \times 90 \text{ days} = \$40.50$

13. Last year a family paid \$33,600 for house payments. If this is equal to 30% of the family's yearly income, the yearly income was \$33,000. True or False?

True is incorrect.  $.3x = \$33,600$ ;  $x = \$112,000$ .

False is correct. This is the case where annual house payment was \$33,000; \$110,000;  $.3x = \$33,000$ .

14. A farmer sold  $62 \frac{1}{2}$  acres of a 750 acre farm. Express as a fraction what part of the farm was sold?

- A. Incorrect.  $62 \frac{1}{2}$  divided by 750 yields  $\frac{1}{12}$ .  $\frac{1}{16}$  would be 46.875 acres.
- B. **Correct.**  $62 \frac{1}{2}$  divided by 750 yields  $\frac{1}{12}$ .
- C. Incorrect.  $62 \frac{1}{2}$  divided by 750 yields  $\frac{1}{12}$ .  $\frac{1}{3}$  would be 250 acres.
- D. Incorrect.  $62 \frac{1}{2}$  divided by 750 yields  $\frac{1}{12}$ .  $\frac{1}{8}$  would be 93.75 acres.

15. Ann and Mary invested \$86,000 in purchasing a home. Mary invested \$9,000 more than Ann. How much did Ann invest?

- A. **Correct.**  $(x + \$9,000 + x) = \$86,000$ ;  $2x = \$77,500$ ;  $x = \$38,500$ .
- B. Incorrect. This is the amount of  $(\$86,000 - \$9,000)$ .
- C. Incorrect. This is the total investment.
- D. Incorrect. This is the amount  $(\$86,000/2)$ .

16. What is the yield (annual return on the investment) for a house that has a market value of \$60,000 and rents for \$500 per month?

- A. Incorrect. 10 is the gross rent multiplier:  $\$60,000/\$6,000 = 100$ .
- B. **Correct.** Yield = Annual rent/market value =  $(\$500 \times 12)/\$60,000 = \$6,000/\$60,000 = 0.10 = 10\%$
- C. Incorrect. This is the monthly return =  $\$500/\$60,000 = 0.0083 = 0.83\%$
- D. Incorrect. 120 is the gross rent multiplier, based on a monthly income.